MOVING CHARGES AND MAGNETISM

ABSTRACT. THIS UNIT(MAGNETIC EFFECTS OF CURRENTS AND MAGNETISM) CARRIES 8 MARKS AND HAS TWO CHAPTERS MOVING CHARGES AND MAGNETISM AND MAGNETISM AND MATTER

1. MAGNETIC FIELD

Evidence of relationship between Electric current and magnetic field

- OERSTED'S EXPERIMENT:When an electric current is passed through a wire a magnetic needle kept near by deflected.The deflection of the needle is always tangential to an imaginary circle drawn around the wire as shown in the diagram below.

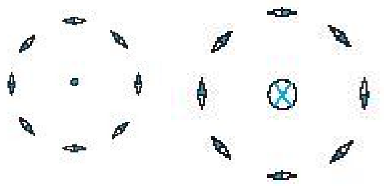


FIGURE 1. *The magnetic field due to a long straight current carrying conductor-with currents flowing perpendicular to the plane of the paper outward and inward-the direction of the field lines also change-*

- When a current is passed through a WIRE passing through a paper (Perpendicular to the plane of the paper) and if iron filings are placed on the paper the iron filings form concentric circles each circle representing the field line.as shown in fig(1.2)

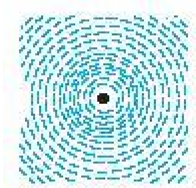


FIGURE 2. *Iron filings arranged due to magnetic field of the wire*

MAGNETIC FIELD $\vec{B}$ Static Charges produce Electric field and moving charges produce magnetic field.Magnetic field is a Vector and is represented using the letter B.
LORENTZ FORCE:If a point charge q moves with a velocity $\vec{V}$ in the presence of both the electric field $\vec{E}$ and magnetic field $\vec{B}$ then the force on the charge q is given by:-

$$\vec{F} = q\,\vec{E} + q(\vec{V} \times \vec{B}) \tag{1}$$

If only Magnetic field is present then the Lorentz force is written as:-

$$\vec{F} = q(\vec{V} \times \vec{B}) \tag{2}$$

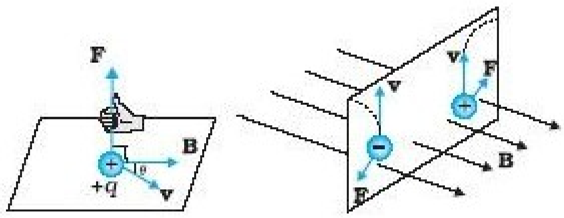
the magnitude of $\vec{F}$ can be written as

$$F = qVBSin\theta \tag{3}$$

and this force $\vec{F}$ is called the magnetic Lorentz force.

FEATURES OF LORENTZ FORCE

- The direction of the Lorentz force for positive charge is opposite to that of negative charge
- The point charge q does not experience any force if the charge is stationary $\vec{F}=0$, if $\vec{V}=0$ refer equation2.
- If the charged particle is moving along the direction of the magnetic field then $\vec{V}$ and $\vec{B}$ are parallel or anti parallel then $\theta = 0$ or 180 hence $sin\theta = 0$ therefore $\vec{F}=0$ hence the particle is undeflected by the magnetic field.(refer Equation 3)
- The Lorentz force acts Perpendicular to the plane containing $\vec{V}$ and $\vec{B}$ in a direction given by the **Right hand Thumb rule** (Curl the fingers of your right hand from $\vec{V}$ toward s $\vec{B}$ (the fingers are curled from $\vec{V}$ toward s $\vec{B}$ because $\vec{V}$ comes first in equation 2) ,the direction in which the thumb points is the direction of the force, $\vec{F}$)-THIS IS FOR POSITIVE CHARGES. If the charge is Negative then curl your right fingers from $\vec{B}$ toward s $\vec{V}$ *(because* $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$*)*



- From the Lorentz equation the defining equation for $\vec{B}$ and hence the unit and dimension of the magnetic field can be determined

$$B = \frac{F}{qV}$$

the corresponding units can be written as

$$B \equiv \frac{Newton.second}{Coulomb.meter}$$

In SI the unit of B is Tesla and

$$1Tesla \equiv \frac{Newton.second}{Coulomb.metre}.$$

Magnetic force on a current carrying Conductor

Consider a conductor of length l and area of cross-section A,let n be the free electron density of the conductor(number of free electrons/unit volume)the free electrons are the mobile charge carriers. For a steady current I flowing in a conductor each mobile charge carrier has drift velocity $\vec{V_d}$,If $\vec{B}$ is the magnetic field then,

Volume of the conductor$=Al$
Total no of free electrons $N = nAl$

The total charge of all the free electrons $q = -nAle$
(where 'e'is the charge of the electron). Using $q = -nale$ in the Lorentz equation $\vec{F} = q(\vec{V} \times \vec{B})$ we get
$\vec{F} = -nAle(\vec{V_d} \times \vec{B}) = -nAle\, \vec{V_d} \times \vec{B}$
but $I = -nAeV_d$ (The negative sign indicates that the conventional current direction and the drift velocity of the electrons are opposite.) hence the force acting on the current carrying conductor is given by:-

$$\vec{F} = I\, \vec{l} \times \vec{B} \tag{4}$$

Here it should be noted that l is a vector which has the same direction as the current I.
The magnitude of the force on the conductor is given by

$$F = BIl \sin\theta \tag{5}$$

The direction of the force acting on the conductor can be found by the Flemings Left Hand rule[FBI].

MOTION OF A CHARGED PARTICLE IN A MAGNETIC FIELD

Consider a charged particle of charge q in a uniform magnetic field $\vec{B}$ With a velocity $\vec{V}$ let $\vec{V}$ be perpendicular to $\vec{B}$.The Lorentz force(2) $\vec{F}$ acts as centripetal force and makes the particle move in a circle as shown in the figure.

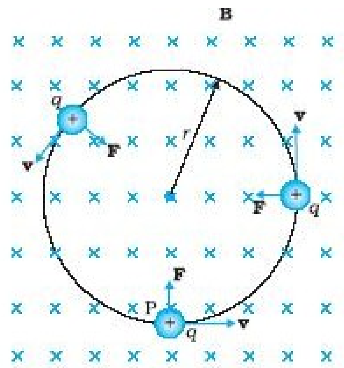


FIGURE 3. *Path of a charged particle in a magnetic field*

Thus ***the charged particle will will move in a circle if $\vec{V}$ and $\vec{B}$ are perpendicular to each other.***Equating the centripetal force(mV^2/r) and the Lorentz force we get

$$BqV \sin\theta = mV^2/r,$$

but $\theta = 90$ hence

$$BqV = mV^2/r \tag{6}$$

From the above equation we can arrive at the radius of the circular path described by the particle as

$$r = mV/Bq \tag{7}$$

Frequency of revolution of the charged particle:
If ω is the angular velocity of the particle and if f is the frequency of revolution of the charged particle then

$$V = r\omega$$
(relation between linear and angular velocity)
and
$$\omega = 2\pi f$$

substituting the radius r from 7 we get

$$\omega = 2\pi f = Bq/m$$

or

$$f = Bq/2\pi m \tag{8}$$

KEY FEATURES OF MOTION OF A CHARGED PARTICLE IN A MAGNETIC FIELD

- The magnetic Lorentz force $\vec{F}$ is perpendicular to the velocity of the particle $\vec{V}$ hence the displacement of the charged particle $\vec{d}$ is also perpendicular to $\vec{F}$,hence the work done by the Lorentz force is"0" ($W = \vec{F} \cdot \vec{d} = Fdcos\theta = Fdcos90 = 0$)
- The magnitude of the velocity also does not change due to the Lorentz Force.
- The only thing that is altered by the Lorentz force is the direction of the charged particle
- The rotational frequency of the charged particle is independent of the energy of the particle or the radius of the circle described.This has important practical application in the design of cyclotron

HELICAL MOTION OF CHARGED PATICLES

When a charged particle is fired in such away that $\vec{V}$ makes an angle to the magnetic field $\vec{B}$, then the velocity vector will have two components one along the magnetic field $\vec{V_B}$ and the other perpendicular to the magnetic field $\vec{V_\perp}$.The component $\vec{V_B}$ remains unaffected by the magnetic field because the angle between $\vec{B}$ and $\vec{V_B}$, $\theta = 0$,hence the Lorentz along $\vec{V_B}$ is also $0(F = qV_B BSin\theta = 0)$.Thus $\vec{V_B}$ provides a forward motion(along $\vec{B}$) to the particle,while the perpendicular component$\vec{V_\perp}$ is affected by the Lorentz force and makes the particle go in a circle,this combined motion gives the particle a **HELICAL PATH**.

PITCH OF THE HELIX

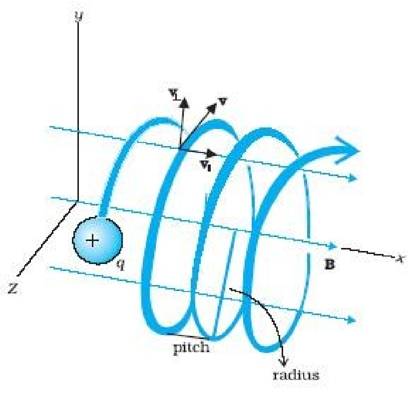


FIGURE 4. *Helical Path Taken by a charged particle when the particle is fired at an angle to magnetic field*

The distance moved by the particle in one rotation along the magnetic field is called the pitch.The time taken by the particle in one rotation is given by time period T, from equation 8 we get

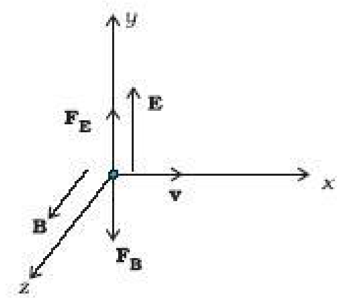
$$T = \frac{1}{f} = \frac{2\pi m}{Bq}$$

$$Pitch\, P = V_B T = V_B.\frac{1}{f} = \frac{2\pi m V_B}{qB} \tag{9}$$

VELOCITY SELECTOR(MOTION OF CHARGED PARTICLE IN COMBINED ELECTRIC AND MAGNETIC FIELD) Consider a charged particle$q$ moving with a velocity $\vec{V}$ in the presence of both electric field$\vec{E}$ and magnetic field $\vec{B}$then the Lorentz force experienced by the particle is given by

$$\vec{F} = q\,\vec{E} + q(\vec{V} \times \vec{B}) = \vec{F_E} + \vec{F_B}$$

Where $\vec{F_E}$ is the electric force and $\vec{F_B}$ the magnetic force if $\vec{E}$ and $\vec{B}$ are perpendicular to the velocity as shown below then



$$\vec{E} = E\hat{j}$$
$$\vec{B} = B\hat{k}$$
$$\vec{V} = V\hat{i}$$
$$\vec{F} = \vec{F_E} + \vec{F_B} = qE\hat{j} + (qV\hat{i} \times B\hat{k}) \tag{10}$$

but $(qV\hat{i} \times B\hat{k} = -qVB\hat{j})$ (because $\hat{i} \times \hat{k} = -\hat{j}$ and $\theta = 90, Sin 90 = 1$) therefore

$$\vec{F} = q(E - VB)\hat{j} \tag{11}$$

From the above equation it is clear that the electric force(qE) and the magnetic force(qVB) are opposite to each other.If we adjust B and E such that the net force(F) on the particle is 0 then

$$F = 0 = q(E - VB)$$

$$V = \frac{E}{B} \tag{12}$$

The physical meaning of this equation is that if a particle has a velocity V which is equal to$\frac{E}{B}$then the particle goes undeflected. ***This condition can be used to select charged particles of a particular velocity out of a beam containing charges moving with different velocities***.

CYCLOTRON.

A Cyclotron is a device used to accelerate charged particles or ions to very high energies.

Principle The principle behind the cyclotron is that the frequency of revolution of a charged particle in a magnetic field is independent of the energy,speed or radius of orbit of the charged particle. Construction The cyclotron has two semicircular disc shaped containers D_1 and D_2 called the Dees.The schematic diagram is shown below(next page).

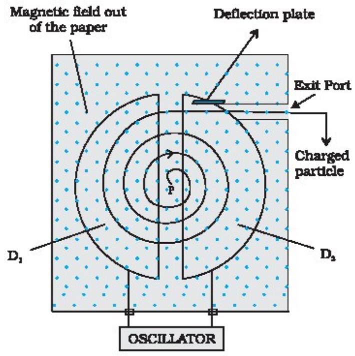


FIGURE 5. Schematic diagram of Cyclotron

A Magnetic field acts perpendicular to the plane of the Dees.The Dees are connected to a high frequency oscillator,and hence an alternating electric field exists between the dees . The whole assembly is kept in an evacuated chamber to minimise collisions with air molecules. Working Inside Dees the magnetic field makes the charged particle go around in a circular path the electric field which acts between the Dees makes the particle gain energy.The sign of the electric field is changed alternatively in tune with the circular motion of the particle. The positive ion or the positive particle which is to be accelerated is released from the center. The frequency of the applied voltage is so adjusted so that the polarity of the Dees is reversed in the same time it takes the particles to complete one half of the revolution or the frequency of the applied voltage has to be the same as the cyclotron frequency the cyclotron frequency is given by.

$$f_c = \frac{Bq}{2\pi m} \tag{13}$$

(Derived from equation 8) The applied frequency(f_a) should be equal to the cyclotron frequency(f_c)this is called the resonance condition.

ENERGY OF THE PARTICLE

Potential Energy Each time the particle crosses the gap between the dees the particle gains an energy qV or if the particle goes around the circle once it gains an energy $2qV$ or if the particle completes n rotations then the particle gains $2nqv$ here V is the potential difference between the Dees.

KINETIC ENERGY from Equation 7 the velocity of the charged particle is

$$V = \frac{BqR}{m}$$

substituting V in the KE equation

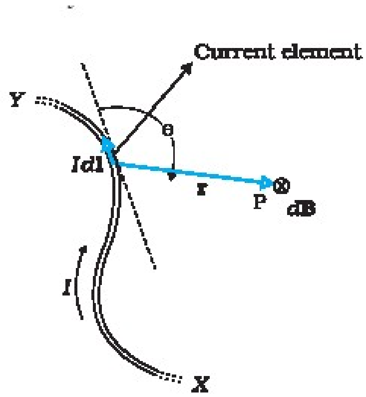
$$KE = \frac{1}{2}mV^2$$

$$KE = \frac{B^2q^2R^2}{2m} \tag{14}$$

This is the maximum um kinetic energy of the particle when it comes out of the cyclotron,and R is the maximum radius the radius of the Dees.

Applications (a)The cyclotron is used to bombard nuclei with energetic particles which are accelerated by the cyclotron.(b)It is used to implant ions into solids and modify their properties (c)In hospitals it is used to produce radioactive substances.

BIOT-SAVARTS LAW The figure below shows a finite conductor carrying a current I



consider a very small element $\vec{dl}$ of the conductor then according to the Biot-Savarts law the magnitude of the magnetic field $\vec{dB}$ due to the element is proportional to the current I the the length of the element dl and inversely proportional to the square of the distance r.Its direction is perpendicular to the plane containing the $\vec{r}$ and $\vec{dl}$ its sense(inward ⊗ or outward ⊙ is given by the right hand thumb rule).Mathematically

$$\vec{dB} \, \alpha \frac{I \, \vec{dl} \times \hat{r}}{r^2}$$

$$\vec{dB} = \frac{\mu_o}{4\pi} \frac{I \, \vec{dl} \times \hat{r}}{r^2} \tag{15}$$

the magnitude of $\vec{dB}$ is given by

$$dB = \frac{\mu_o}{4\pi} \frac{I dl \sin\theta}{r^2} \tag{16}$$

$\frac{\mu_o}{4\pi}$ is the constant of proportionality and μ_o is called the permeability of free space and $\frac{\mu_o}{4\pi} = 10^{-7}$ Tm/A

Comparison between Coulombs law in electrostatics and Biot-Savarts Law

- ☞ Both are long range forces
- ☞ The magnetic field is produced by a vector field $I \, \vec{dl}$,whereas the electric field is produced by a scalar source the charge q
- ☞ The direction of the electrostatic field is along the displacement vector joining the source and the point for whereas the magnetic field is perpendicular to the plane containing $\vec{dl}$ and $\vec{r}$.

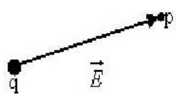
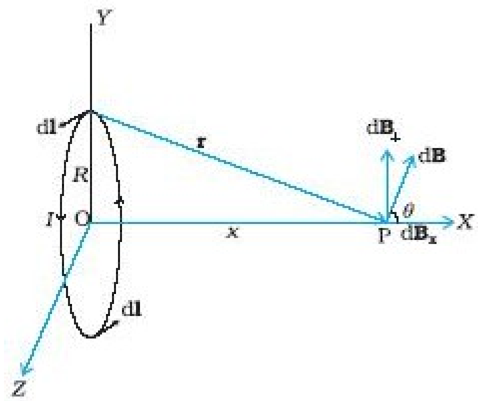


FIGURE 6. *the direction of the Electric field is along the line joining the charge the point*

☞ The superposition principle applies for both
☞ There is an angle dependence in Biot-savarts law but not so in coloumbs law(There is a "$sin\theta$" inthe Biot Savart's law).

MAGNETIC FIELD ON THE AXIS OF A CIRCULAR CURRENT CARRYING LOOP

Consider a circular current loop of radius R carrying a current I,the coil is placed in the $y-z$ plane with the center of the loop at the origin as shown below



The magnetic field at the point p at a distance x can be calculated as below:-
The magnitude of the magnetic field due to the element dl is given by the equation 16

$$dB = \frac{\mu_o}{4\pi}\frac{Idl\sin\theta}{r^2}$$

but $\theta = 90$ and $r^2 = x^2 + R^2$,hence

$$dB = \frac{\mu_o}{4\pi}\frac{Idl}{(x^2 + R^2)} \tag{17}$$

The direction of $\vec{dB}$is perpendicular to the plane formed by $\vec{dl}$ and $\vec{r}.\vec{dB}$ is perpendicular to the plane containing r and $\vec{dB}$ as shown in the diagram.$\vec{dB}$ has two components $dB_\perp$(perpendicular to the X-axis) and dB_x(along the X-axis)

$$dB_\perp = dB\sin\theta$$

$$dB_x = dB\cos\theta$$

There are infinite number of dl elements in the loop the sum of all the perpendicular components produced($dB_\perp = dB\sin\theta$) by all the dl elements adds up to 0 because they are opposite to each other.Hence the net magnetic field is found by summing up all the x components of dB i.e db_x.This is done by integrating $db_x = dBcos\theta$ over the full length of the loop($2\pi r$)

$$\oint dB_x = \oint dBcos\theta \tag{18}$$

substituting $cos\theta = \frac{R}{(x^2+R^2)^{1/2}}$ and equation 17 in 18 we get

$$\oint dB_x = B_x = \oint \frac{\mu_o}{4\pi}\frac{Idl}{(x^2 + R^2)}\frac{R}{(x^2 + R^2)^{1/2}}$$

$$= \oint \frac{\mu_o}{4\pi} \frac{IdlR}{(x^2 + R^2)^{3/2}} = \frac{\mu_o}{4\pi} \frac{IR}{(x^2 + R^2)^{3/2}} \oint dl$$

but $\oint dl = 2\pi R$. Thus the magnetic field due to the entire loop at the point P is

$$\vec{B} = B_x \hat{i} = \frac{\mu_o}{2} \frac{IR^2}{(x^2 + R^2)^{3/2}} \hat{i} \tag{19}$$

Special Cases

Magnetic field at the center of the loop:- Here $x = 0$ hence equation 19 becomes

$$\vec{B_c} = \frac{\mu_o I}{2R} \hat{i} \tag{20}$$

When $x >> R$

here R can be neglected hence we get

$$\vec{B} = \frac{\mu_o I R^2}{2x^3} \hat{i} \tag{21}$$

MAGNETIC FIELD LINES of the circular loop

The field lines are as shown below

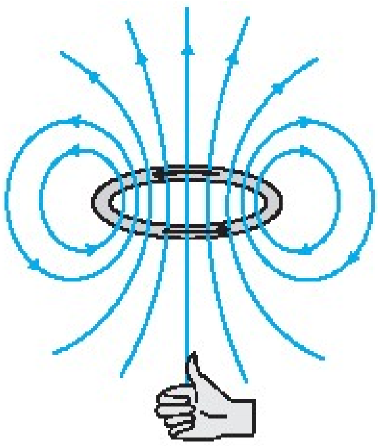


FIGURE 7. *the direction of the magnetic field due to a circular loop depends on the direction of the current*

To find the direction of the magnetic field the right hand thumb rule is used curl the fingers(right hand) in the direction the current flows the direction in which the thumb points is the direction of the magnetic field.

noteA current carrying loop acts as a magnetic dipole the face of the coil from which the lines of force seem to come out is the north pole and the face into which the lines seem to enter acts as the south pole.

AMPERE S CIRCUITAL LAW

The law states that the the line integral of the magnetic field around any closed path is equal to μ_o(absolute permeability of free space)times the total current I_e threading the closed loop

$$\oint \vec{B} \bullet \vec{dl} = \mu_o I_e \tag{22}$$

Magnetic field due to a solenoid

A solenoid is a tightly wound helical loop formed with an insulated wire such that the length of the solenoid is much larger than its diameter.

Consider a solenoid of n turns per unit length.When a current i is passed through the solenoid a magnetic field is produced.The The field at the interior of the solenoid is strong and uniform,whereas the field outside is weak(≈ 0).
To determine the magnetic field of the of the solenoid ,consider a rectangular amperean loop ABCD of length L as shown below

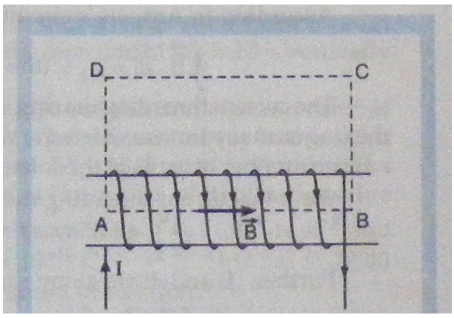


FIGURE 8. *Amperean loop for solenoid*

applying the amperes circuital law along all sides of the rectangle we get

$$\oint_{ABCB} \vec{B}\,.\,\vec{dl} = \int_{AB} \vec{B}\,.\,\vec{dl} + \int_{BC} \vec{B}\,.\,\vec{dl} + \int_{CD} \vec{B}\,.\,\vec{dl} + \int_{DA} \vec{B}\,.\,\vec{dl} = \mu_o I_e$$

$$\int_{BC} \vec{B}\,.\,\vec{dl} = \int_{DA} \vec{B}\,.\,\vec{dl} = 0$$

This is because in the segments BC and DA $\vec{dl} \perp \vec{B}$ hence $\vec{B}\,.\,\vec{dl} = Bdlcos\theta = 0(\theta = 90)$.
Similarly

$$\int_{cd} \vec{B}\,.\,\vec{dl} = 0$$

This is because the field outside the solenoid is 0.
Thus

$$\oint_{ABCB} \vec{B}\,.\,\vec{dl} = \int_{AB} \vec{B}\,.\,\vec{dl} = \int_{AB} Bdlcos0 = \int_{AB} Bdl = \mu_o I_e$$

$$= \int_{AB} Bdl = B\int_{AB} dl = BL = \mu_o I_e \tag{23}$$

(because $\int_{AB} dl = L$)
If n is the no. of turns per unit length the total no. of turns in the length L is nl.If I is the current through one turn then the current enclosed by the loop $I_e = nLI$,replacing for I_e in equation 23 we get

$$BL = \mu_o nLI$$

or

$$B = \mu_o nI \tag{24}$$

<u>MAGNETIC FIELD DUE TO A TOROID</u>
A Toroid is a hollow circular ring on which a large number of turns of wire are closely wound.It can be viewed as a solenoid turned into a circular ring.Let n be the number of turns per unit length

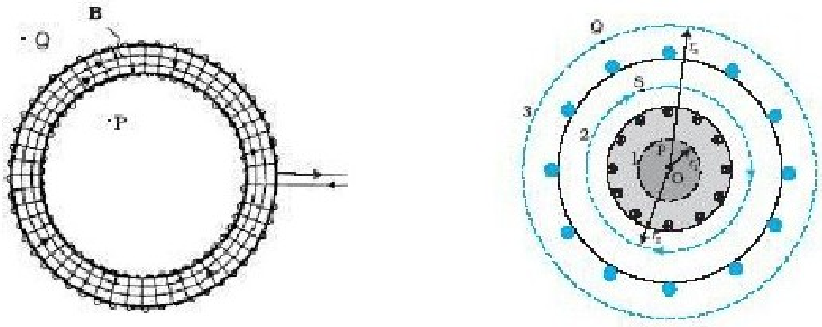


FIGURE 9. *The figure on the left shows the Toroid and on the right the amperean loops chosen-loops1,2 and 3*

To find the magnetic field the amperean loops 1,2 and 3 are chosen as shown above.

loop 1

In loop 1 the current enclosed by Amperean loop 1 is $I_e = 0$ as per amperes circuital law

$$\oint \vec{B} \, . \, \vec{dl} = \mu_o I_e$$

$I_e = 0$,hence $B = 0$
Thus the magnetic field at any point p in the open space inside the toroid is 0.

loop 3

The current enclosed by this loop I_e is also 0 because the net current i.e the current coming out of the plane of the paper and the current going inside the plane of the paper cancel out making the net current $I_e = 0$.Hence at all points Q outside the toroid the magnetic field is 0.

loop 2

The magnetic field inside the toroid at S.
As per Ampere s circuital law

$$\oint \vec{B} \, . \, \vec{dl} = \mu_o I_e$$

$$\oint dl = L = 2\pi r$$

If I is the current that flow s in each turn then the current enclosed in N turns I_e is given by,

$$I_e = NI = nLI = n2\pi rI$$

Substituting for I_e and $\oint dl$ in the Ampere s circuital law we get

$$B(2\pi r) = \mu_o I_e = \mu_o n(2\pi r)I$$

or

$$B = \mu_o nI \tag{25}$$

Thus the magnetic field due to a solenoid and a toroid are the same.

MAGNETIC FIELD DUE TO LONG STRAIGHT WIRE INSIDE AND OUTSIDE

Consider a conductor of radius a carrying a current I,distributed uniformly through the cross section of the conductor

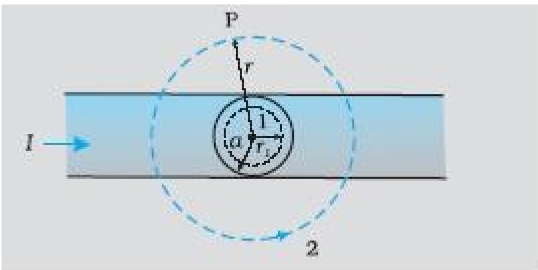


FIGURE 10. *Amperean loops for a wire-loop 2 for field out side the wire and loop1 for field inside*

outside the conductor $r > a$

The Amperean loop labeled 2 is chosen,the loop is concentric with the cross section.For this loop

$$\oint dl = L = 2\pi r$$

and

$$I_e = I$$

applying this in the Ampere s circuital Law we get

$$\oint \vec{B} \bullet \vec{dl} = \mu_o I_e$$

$$B(2\pi r) = \mu_o I$$

or

$$B = \frac{\mu_o I}{2\pi r} \tag{26}$$

Thus for $r > a$

$$B \propto \frac{1}{r}$$

The direction of the magnetic field can be found using the right hand thumb rule "stretch your thumb along the direction of the current in the wire and curl your fingers the direction in which the curl gives the direction of the field"
diagram here

inside the conductor $r < a$

The loop labeled 1 is chosen as the Amperean loop the Ampere s circuital law gives

$$\oint \vec{B} \bullet \vec{dl} = \mu_o I_e$$

$$\oint dl = L = 2\pi r$$

The current enclosed by the Amperean loop is not I but less than I and can be found using the following arguments πa^2 of area carries a current of I Amperes,therefore 1 unit area will carry a current of $\frac{I}{\pi a^2}$ Amperes of current,hence πr^2 of area will carry a current of $\pi r^2(\frac{I}{\pi a^2})$ Amperes of current-that s the current enclosed by the Amperean loop I_eTherefore $I_e = \frac{Ir^2}{a^2}$ using this in the Amperes circuital law we get

$$B(2\pi r) = \mu_o \frac{Ir^2}{a^2}$$

$$B = (\frac{\mu_o I}{2\pi a^2})r \tag{27}$$

Thus for the magnetic field inside the conductor($r < a$) $B \propto r$ Graphically the variation can be represented as

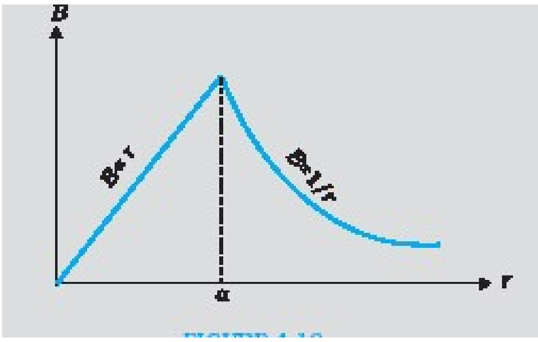


FIGURE 11. *variation of magnetic field due to a wire with distance*

<u>FORCE BETWEEN TWO PARELLEL CURRENT CARRYING CONDUCTORS</u>

The figure below shows two parallel current carrying conductors

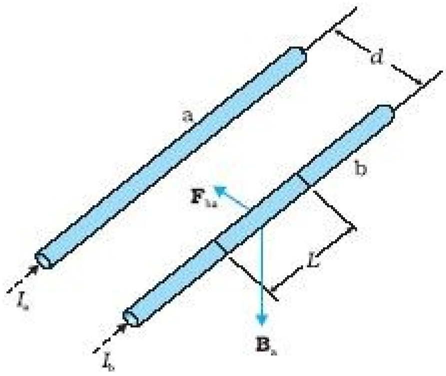


FIGURE 12. *Force between two parellel wires*

Consider two parallel conductors 'a' and 'b' separated by a distance d carrying currents I_a and I_b respectively.The conductor 'a' produces a magnetic field $\vec{B_a}$ and the conductor 'b' produces a magnetic field $\vec{B_b}$.The conductor 'a' is immersed in the magnetic field produced by the conductor b and similarly the conductor 'b' is immersed in the magnetic field produced by 'a'.The magnetic field $\vec{B_a}$ is the same along the length of the conductor 'b' and similarly $\vec{B_b}$ along 'a'.The direction of $\vec{B_a}$ near the conductor 'b' is downwards(use the right hand thumb rule) using equation 26

$$B_a = \frac{\mu_o I_a}{2\pi d}$$

(direction $\otimes$)
due to this field the conductor 'b' will experience a force F_{ba},the force on a segment L of 'b' due to 'a' given by

$$F_{ba} = B_a I_b L = \frac{\mu_o I_a I_b}{2\pi d} L$$

Using the Flemings left hand rule(FBI)the direction of the force F_{ba} is toward s 'a'.Similarly the force F_{ab} can be found as

$$F_{ab} = B_b I_a L = \frac{\mu_o I_b I_a}{2\pi d} L$$

and the direction of F_{ab} is toward s 'b'
since the the forces are equal and opposite

$$\vec{F_{ab}} = -\vec{F_{ab}}$$

Thus

- Two parallel current carrying conductors separated by a distance exert a force on each other
- If the conductors carry current in the same direction then the attract each other
- If the conductors carry current in the opposite direction then they repel each other
- from the equation $F_{ba} = \frac{\mu_o I_a I_b}{2\pi d} L$ one ampere is defined as follows:-The ampere is that value of steady current which when maintained in each of two very long conductors of negligible cross section and placed 1 meter apart in vacuum would produce in each of these conductors a force of 2×10^{-7} newtons per meter of length (i.e put $I_a = I_b = 1A, d = 1m, L = 1m, \frac{\mu_o}{4\pi} = 10^{-7}$ then $F_{ab} = F_{ba} = 2 \times 10^{-7}$)

TORQUE ON A CURRENT CARRYING RECTANGULAR LOOP

Consider a rectangular loop ABCD carrying a current I placed in a uniform magnetic field $\vec{B}$ as shown in the figure below

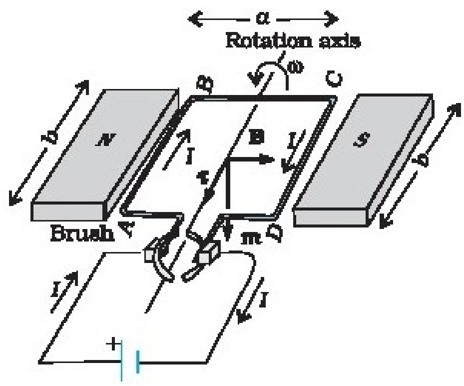
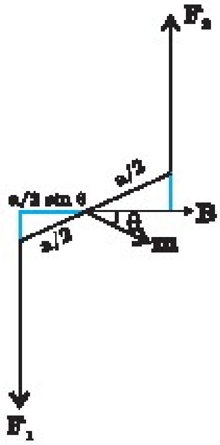


FIGURE 13. *A current loop in a magnetic field experiences a torque-note the direction of $\vec{m}$*

The magnetic field exerts no force on the arms AD and BC(because $\vec{F} = I\vec{l} \times \vec{B}$ or $F = BIlsin\theta$,as θ the angle between $\vec{Il}$ and $\vec{B} = 0 or 180, F = 0$).
The force on arm AB $F_1 = BIb$
The force on arm CD $F_2 = BIb$
The direction of the forces F_1 and F_2 can be found using the Flemings left hand rule(FBI),F_1 is perpendicular to the plane of the paper and in wards and F_2 is perpendicular to the plane of the paper and out wards.

Thus the net translational force on the conductor is 0,this is because the forces $\vec{F_1}$ and $\vec{F_2}$ which act on the lengths AB and CD are equal and opposite,But the net torque acting on the loop is not 0.The figure below shows the view of the loop from AD
figure below.



This torque tends rotate the loop clockwise and its magnitude is given by($\tau = force \times \perp distance$):-

$$\tau = F_1 \frac{a}{2} sin\theta + F_2 \frac{a}{2} sin\theta$$

since $F_1 = F_2 = BIb$

$$\tau = BIb\frac{a}{2} sin\theta + BIb\frac{a}{2} sin\theta$$

$$\tau = BIabsin\theta$$

$$\tau = IABsin\theta \tag{28}$$

where $A = ab$ is the area of the rectangular loop.

<u>MAGNETIC MOMENT $\vec{m}$</u>:-The magnetic moment of a current loop is vector and is defined as

$$\vec{m} = I \vec{A} \tag{29}$$

The direction of $\vec{A}$ and hence the direction of $\vec{m}$ is determined using the right hand thumb rule "curl your fingers along the direction of the current in the loop then the direction in which the thumb points is the direction of $\vec{m}$".If there are N turns in the coil then the magnetic moment is given by:-

$$\vec{m} = NI \vec{A} \tag{30}$$

hence the magnitude of torque (τ), equation 27 can be written as

$$\tau = mBsin\theta \tag{31}$$

In Vector form the equation becomes

$$\vec{\tau} = \vec{m} \times \vec{B} \tag{32}$$

<u>DIPOLE MOMENT OF A REVOLVING ELECTRON</u>

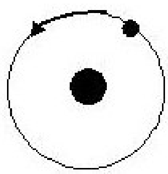


FIGURE 14. *Hydrogen atom model* $\otimes$ μ_l *is inward*

Consider the Bohr model of the hydrogen atom the with the electron revolving around the nucleus. The motion of th electron which is in the counter clock wise direction can be considered as "conventional" current flowing in the clock wise direction(moving charges constitute current).The direction of the magnetic moment is inward($\otimes$) $\perp$ to the plane of the paper(this found by using the right hand thumb rule) as

$$I = \frac{q}{t}$$
$$I = \frac{e}{T} \tag{33}$$

where e is the charge of the electron and T is the time period of the electron.If v is the speed of the electron and r the radius of the orbit,then

$$T = \frac{v}{2\pi r}$$

using the above equation in 33 we get

$$I = \frac{ev}{2\pi r} \tag{34}$$

The magnetic moment associated with the revolving electron is denoted by the symbol μ_l.using equation 29 we get

$$\mu_l = IA = I\pi r^2$$

substituting equation. 34 the above equation we get

$$\mu_l = \frac{evr}{2}$$

multiplying and dividing the RHS of the above equation by m_e the mass of the electron we get

$$\mu_l = \frac{m_e evr}{2m_e} = \frac{e}{2m_e}(m_e vr)$$

in the above equation $m_e vr$ is the angular momentum(l) of the electron.Hence

$$\mu_l = \frac{e}{2m_e}(l) \tag{35}$$

Vectorially the above equation can be written as

$$\vec{\mu_l} = -\frac{e}{2m_e}\vec{l} \tag{36}$$

The negative sign indicates that the angular momentum and the the magnetic moment are opposite($-e$ because electron is negative).
The ratio

$$\frac{\mu_l}{l} = \frac{e}{2m_e}$$

is called the gyromagnetic ratio and has a value of $8.8 \times 10^{10} C/Kg$.
As per Bohr s hypothesis the angular momentum of the electron can take only certain discrete values namely

$$l = \frac{nh}{2\pi}$$

where h is the Planck s constant and n is a natural number $n = 1, 2, 3....$hence equation 35 can be written as

$$\mu_l = \frac{ne}{4\pi m_e}h \tag{37}$$

KEY CONCEPTS

- The elementary dipole moment of the electron ($(\mu_l)_{min}$)can be calculated by putting $n = 1$ in the above equation and substituting the values of e, m_e and h and this value is called the **Bohr magneton** and is found to be

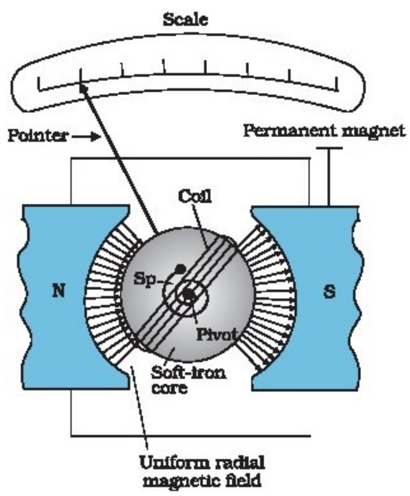
$$(\mu_l)_{min} = 9.27 \times 10^{-24} Am^2$$

.

- All charged particles in uniform circular motion will have a magnetic moment give by equation 35($\vec{\mu_l} = -\frac{e}{2m_e}\vec{l}$) and is called Orbital magnetic moment
- Electrons also have spin magnetic moment and has a value given by $(\mu_l)_{min} = 9.27 \times 10^{-24} Am^2$
- Magnetism in materials like Iron is fundamentally due to the spin magnetic moment

MOVING COIL GALVANOMETER

The moving coil galvanometer is an instrument used to detect current
PRINCIPLEThe moving coil galvanometer works on the principle of torque acting on a rectangular current carrying current loop.
Construction The galvanometer consists of a coil with many turns placed in a magnetic field created by the curved poles of a strong magnet.The coil is wound over a *cylindrical soft Iron core which makes the magnetic field radial and strong.*The coil and the core is pivoted with the help of a spring S_p,this spring provides counter torque to the torque produced by the current carrying loop.



Working When a current is passed through the rectangular loop the loop experiences a torque given by

$$\tau = NIAB\,sin\theta$$

Since the field is radial $\theta = 90$ hence

$$\tau = NIAB$$

The spring S_p gives a counter torque

$$\tau = K\phi$$

where K is the spring constant and ϕ is the angle by which the needle deflects, equating the above two equations we get

$$k\phi = NIAB$$

or

$$\phi = \left(\frac{NAB}{K}\right) I \tag{38}$$

Thus the angle by which the needle deflects is proportional to the current

$$\phi \propto I$$

Conversion of Galvanometer into AMMETER The galvanometer cannot be used to measure current directly for the following two reasons:

- The Galvanometer is a very sensitive instrument and cannot measure large currents.
- For measuring currents, the galvanometer has to be connected in series, and as it has a large resistance, this will change the value of the current in the circuit.

To overcome these difficulties,a small resistance r_s,called shunt resistance, in parallel with the galvanometer coil; so that most of the current passes through the shunt.

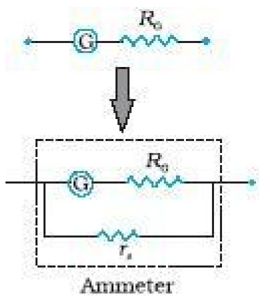


FIGURE 15. *to convert a galvanometer to Ammeter a shunt is connected*

If R_G is the resistance of the galvanometer and r_s is the resistance of the shunt.If I_G is the maximum current that a galvanometer can handle or the current which gives full scale deflection in the galvanometer and if I is the current then as R_G and r_s are parallel the potential drop is the same hence

$$I_G R_G = (I - I_G) r_s$$

,hence

$$r_s = \frac{I_G R_G}{(I - I_G)} \tag{39}$$

Conversion of Galvanometer into VOLTMETER

The galvanometer can also be converted into a voltmeter and used as a voltmeter to measure the voltage across a given section of the circuit. For measuring voltage:

- The voltmeter must be connected in parallel with that section of the circuit across which the Potential difference is to be measured .
- It must draw a very small current, otherwise the voltage measurement will disturb the original set up by an amount which is very large.

Let the voltage to be measured be V and the current which is produces maximum deflection be I_G. To keep the disturbance due to the measuring device low, a large resistance R is connected in series with the galvanometer. This arrangement is schematically depicted below The resistance R which is to be attached in series is given by

$$V = I_G R_G + I_G R$$

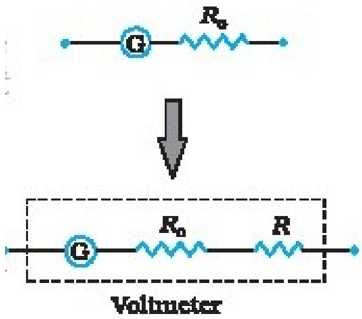


FIGURE 16. *To convert a galvanometer to voltmeter a high resistance is connected in series*

or

$$R = \frac{V - I_G R}{I_G} \tag{40}$$

Voltage Sensitivity: Voltage sensitivity(v_s) is the deflection per unit voltage.and is given by

$$V_s = \frac{\phi}{V}$$

using ϕ from equation 38 we get

$$V_s = \frac{\phi}{v} V_s = \frac{\left(\frac{NAB}{K}\right) I}{V} = \left(\frac{NAB}{K}\right) \frac{1}{R} \tag{41}$$

- ***The voltage sensitivity of a voltmeter can be increased by increasing the magnetic field and decreasing resistance of the coil.The number of turns cannot be increased because doing so will increase the resistance of the coil,in accordance with equation 41.***

Current Sensitivity:- Voltage sensitivity(v_s) is the deflection per unit current.and is given by

$$I_s = \frac{\phi}{I}$$

$$I_s = \frac{\left(\frac{NAB}{K}\right) I}{I} = \frac{NAB}{K} \tag{42}$$

- ***The current sensitivity of an ammeter can be increased by increasing the magnetic field and decreasing the "spring constant k" in accordance with equation 42***

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